Marginal Stockholders and Implied Tax Rates

LeRoy D. Brooks
John Carroll University, lbrooks@jcu.edu

Charles E. Edwards
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MARGINAL STOCKHOLDERS AND IMPLIED TAX RATES

LeRoy D. Brooks and Charles E. Edwards*

In this review some years ago, Edwin Elton and Martin Gruber (1970) used the concepts of market equilibrium and differential tax rates between capital gains and dividend income to structure a theoretical model which they then used empirically to estimate marginal stockholder tax rates. Their specification of an equilibrium condition is appropriate for a security seller who qualifies for preferential tax treatment of capital gains. However, their assumption that such a stockholder is the marginal stockholder in a market equilibrium is questionable. This assumption, along with disregard of transactions and other costs, is essential to their empirical derivation of stockholder tax rates. This note presents an alternative explanation of their empirical results.

The Elton and Gruber Model

As specified by Elton and Gruber (E & G), a stockholder is indifferent to the timing of his sale of shares if his expected wealth is not affected by the transaction. Indifference results when the net after-tax proceeds from the sale of shares before they go ex-dividend equals the combination of after-tax dividend and after-tax stock sale proceeds obtained by not selling the shares until the ex-dividend date. In this case,

$$PB - tc(PB - PC) = PA - tc(PA - PC) + D(1 - to)$$

(1)

where

- $PB =$ price on the day before the stock goes ex-dividend,
- $PA =$ price of the stock on the ex-dividend day,
- $PC =$ price at which the stock was purchased,
- $tc =$ tax rate on capital gains,
- $to =$ tax rate on ordinary (dividend) income, and
- $D =$ amount of the dividend.

Rearranging (1) they obtain

$$\frac{PB - PA}{D} = \frac{1 - to}{1 - tc}$$

(2)

from which the statistic $(PB - PA)/D$ represents the ex-dividend pricing behavior which causes a stockholder with a particular set of tax rates $tc$ and $to$ to be indifferent to the timing of the sale of stock.

Explicit in their model development and interpretation of the statistic $(PB - PA)/D$ is the assumption that the marginal sellers of stock qualify for long-term capital gains tax treatment.

In a rational market the fall in price on the ex-dividend day should reflect the value of dividends vis-à-vis capital gains to the marginal stockholders. Since dividends and capital gains are taxable at different rates, the relative tax rate on these two types of income affect the decision. (E & G, 1970, p. 69)

The assertion that capital gains and dividends of the marginal stockholder are taxed differentially is maintained in their empirical analysis to derive implied tax rates. For the period covered by their empirical study, they observe that $to$ is one half of $to$ or 25%, whichever is lower. From this assumption and $(PB - PA)/D$ statistics, they empirically derive implied tax rates of the marginal stockholder both for their entire sample and for various dividend yield classes arranged by deciles.

Model Extensions

Unless entry into the market equilibrating process is restricted, an assumption that the marginal stockholder has preferential long-term capital gains tax treatment may be unsound. The "equilibrium" implied by the assumed preferential tax treatment of capital gains permits possible increases in wealth for non-preferentially taxed investors. In a pre- to post-dividend-record-date-decision environment, long-term capital gains stockholders prefer a dollar of capital gains to a dollar of dividends because of the differential tax advantage of capital gains. Since investors lose a portion of their preferential long-term capital gains tax shield at the ex-dividend date, the statistic $(PB - PA)/D$ is expected to be less than one if long-term capital gains sellers establish the market equilibrium since, in equation (2), $tc < to$.

Unlike long-term capital gains shareholders, ordinary income (short-term capital gains) and tax free shareholders do not lose preferential tax shields when making decisions to sell shares relative to the timing of the ex-dividend date. Ordinary income and tax free shareholders are possible marginal entrants since they may increase their wealth under pricing conditions which would otherwise result if only long-term capital gains shareholders entered the market equilibrating process. Two principal classes of potential sellers are now considered.¹

¹ Other potential entrants might also impact upon the equilibrium statistic $(PB - PA)/D$. The impact of potential purchasers is considered in the appendix.
Non-preferentially Taxed, Sunk-Cost Sellers

The ordinary income or tax free (short- or long-term) seller for whom \( P_C \) is also a sunk cost is very similar to the long-term preferentially-taxed sunk-cost seller postulated for equation (2) (for whom \( t_e < t_o \)). The non-preferentially taxed seller, however, is one whose transaction either has no tax effects (\( t_e = t_o = 0 \)) or whose purchase of stock was not sufficiently remote in time to qualify for preferential capital gains tax treatment (\( t_e = t_o \)). In a perfect market, with entry of non-preferentially taxed sunk-cost stockholders, the expected equilibrium statistic is

\[
(P_B - P_A)/D = 1.
\]

(3)

Since \( t_e = t_o \), the equilibrium condition in this case is invariant to tax rates. Although the long-term capital gains seller requires the pricing behavior \((P_B - P_A)/D < 1\) for indifference, the non-preferentially taxed seller may be expected to enter the market up to a point where \((P_B - P_A) = D\). Thus, the pricing behavior toward which the long-term capital gains seller is indifferent is not a matter of indifference to the non-preferentially taxed seller.

For example, let \( t_e = 36.5\% \) and \( t_o = 18.25\% \), which correspond closely to the rates derived by E & G from their empirically observed statistic \((P_B - P_A)/D = .7767\). In an E & G proposed equilibrium, equation (2), a stock with \( P_B = 40 \) and \( D = .50 \) has in equilibrium \( P_A = 39.61 \). A non-preferentially taxed shareholder, having already decided to sell shares, must now determine the more appropriate timing of sale. He can either sell his shares at \( P_B = 40 \) or wait to close out his position the following day at \( P_A = 39.61 \), thereby also obtaining \( D = .50 \). By postponing sale one day, wealth is increased by \$1.11 per share for the tax free institution or \$0.07 per share after taxes for the ordinary income short-term shareholder. The potential gain is greater at higher dividend yields, as illustrated in table 1 under hypothetical E & G assumptions of equilibrium conditions.

Since non-preferentially taxed sellers can increase their wealth under the E & G specified equilibrium of equation (2), the specification for indifference based on long-term capital gains sellers may not represent a final market equilibrium. Non-preferentially taxed sellers entering the market can effect a new equilibrium price, being willing to accept a lower \( P_A \) relative to \( P_B \), thereby displacing long-term sellers at the margin.

The Arbitrager

Another possible, and potentially very important, entrant into the equilibrating process is the arbitrager seeking to profit from buying at \( P_B \) and selling at \( P_A \) while also receiving \( D \). For entry of a short-term arbitrager, a \((P_B - P_A) < D\) differential must be sufficient to cover "in and out" transactions costs and tax effects. If the equilibrium pricing behavior were as specified in equation (2) with \( t_e < t_o \), the short-term arbitrager potentially could obtain a gain, \( G \), which must also cover his opportunity costs of investment. Since \( t_e = t_o \) for a short-term buyer and seller, the required return for entry is

\[
G(1 - t_e) = P_A(1 - t_o) - t_o(P_A(1 - t_e) - P_B(1 + t_o)) + D(1 - t_o) - P_B(1 + t_o)
\]

(4)

where \( t_o \) is the effective rate of transactions cost relative to selling price per share. The cost may be viewed as that which applies to an economic order quantity, probably round lots, and may include both fixed and variable components. Equation (4) reduces to

\[
\frac{P_B - P_A}{D} = 1 - \frac{t_o(P_B + P_A) + G}{D}.
\]

(5)

Ordinary investors are unlikely to have sufficiently low transactions costs to encourage their participation in the market equilibrating process. However, the potential entry of "floor traders," who, as Exchange members, pay reduced brokerage commissions on purchases and sales on their own account, can not easily be ruled out. Their transactions costs are near zero, although they pay Federal and State transfer taxes usually amounting to a few cents on shares they sell. Their entry at the margin is consistent with an expected statistic \( (P_B - P_A)/D < 1 \) to cover any required transactions and opportunity costs. Thus, an empirical finding of \( (P_B - P_A)/D < 1 \) is not necessarily a function of \( t_e \) and \( t_o \), since, again, there is no differential tax on capital gains versus ordinary income.

Impact of Additional Entrants

Markets in reality may be less than perfect. A possible set of conditions may be hypothesized to support using the statistic \( (P_B - P_A)/D \) to derive estimates of marginal stockholder tax rates. Sound estimates of tax rates might be derived if non-preferentially taxed sellers do not displace long-term sellers at the margin in a market equilibrating process. This might happen if

\begin{table}
<table>
<thead>
<tr>
<th>Dividend Yield</th>
<th>Annual Gain in Wealth as a Percentage of Share Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>.06%</td>
</tr>
<tr>
<td>3</td>
<td>.17</td>
</tr>
<tr>
<td>5</td>
<td>.28</td>
</tr>
<tr>
<td>8</td>
<td>.45</td>
</tr>
<tr>
<td>10</td>
<td>.56</td>
</tr>
</tbody>
</table>

*Where column two is for a tax-free institution and column three is for an ordinary income short-term shareholder.*
1. non-preferentially taxed investors do not have an adequate number of shares or capital to effect a final equilibrium,
2. the expected return differential, \((P_A + D) - P_B\), is insufficient to cover the associated costs of entry,
3. the potential non-preferentially taxed entrants are operating nonoptimally (inefficient or imperfect markets),
4. market or Exchange trading rules prohibit sufficient entry of floor traders to materially affect equilibrium, or
5. by coincidence the costs of other classes of entrants, such as floor traders, result in the same equilibrium pricing.

Transactions costs, as noted previously, are likely to discourage entry of ordinary investors as arbitragers; but floor traders whose transactions costs are minimal may not be similarly discouraged unless their opportunity costs are high. Perhaps more importantly, non-preferentially taxed sunk-cost sellers (including perhaps some large institutions) seem no more likely to be excluded by the above conditions than long-term capital gains sellers. Some of the above conditions might be influential in less than perfect markets, but probably no more so for non-preferentially taxed sellers than for long-term preferentially taxed sellers. The end result may be a mixture of long- and short-term entrants into the market equilibrating process.

**Clientele Effects**

In arranging their data by deciles according to dividend yields \((D/P)\), E & G observed a generally increasing statistic \((P_B - P_A)/D\) with increasing yields, which they interpreted as evidence of a tax "clientele effect." The higher the yield, the higher was the statistic, and the lower the implied tax bracket of the marginal stockholder, except for the two highest \(D/P\) deciles for which E & G reported indeterminate tax rates.

The increasing statistic perhaps partly reflects a clientele effect. However, if entry of non-preferentially taxed sellers occurs, an alternative explanation for the increasing statistic is available. From equation (3), a statistic \((P_B - P_A)/D = 1\) is what should be expected in equilibrium with rational non-preferentially taxed sellers at the margin. E & G’s reported statistics for the higher \(D/P\) deciles approach or exceed 1.0 (E & G, 1970, table 3). These results are consistent with the expected entry by non-preferentially taxed investors.2 Perceived gains are potentially greater on higher yielding stocks (as shown in table 1) and are more likely to be deemed worthy of pursuit under real-world conditions of less than perfect knowledge. Thus, the likelihood and extent of non-preferentially taxed sellers’ entry is potentially greater for high \(D/P\) stocks. Hence, the \((P_B - P_A)/D\) statistic may be larger for the higher \(D/P\) deciles because of entry of non-preferentially taxed sellers,3 and not necessarily from a tax-rate clientele effect of long-term preferentially taxed sellers.

**Conclusions**

E & G created an inventive approach to estimating marginal stockholder tax rates. Unfortunately for those who desire a convenient procedure, their estimating procedure is of doubtful accuracy. We would have no reservations regarding use of their methodology for estimating tax rates if we were assured that marginal sellers of all shares in a sample of stocks going ex-dividend were long-term capital gains sellers. However, if some displacement of long-term capital gains sellers results from entry of non-preferentially taxed sellers at the margin for some stocks, the accuracy of their procedure is questionable. Furthermore, if non-preferentially taxed holders were fully to displace long-term capital gains holders as sellers at the margin, no inferences of tax rates can be made, as may be seen in the equilibrium condition of equation (3).

A statistic \((P_B - P_A)/D < 1\) may reflect the influence of a floor trader with transactions and opportunity costs to be covered. If perchance the floor trader is the determining influence at the margin, again no inferences can be made regarding tax rates.

The relationships determining the pricing statistic are most likely more complex than postulated by E & G. Only if potential entrants other than long-term capital gains sellers are fully excluded, which seems unlikely, can accurate statements be made regarding marginal stockholder tax rates using E & G’s methodology. Biased by the probable entry of others besides long-term capital gains sellers into the market equilibrating process, their approach at best may only partially capture the effects of tax rate differentials between ordinary and long-term capital gains income.

It follows that E & G’s summary statements on the cost of retained earnings, stockholder tax-rate clientele effects, and market rationality of high tax bracket shareholders should be viewed with considerable caution. Additionally, caution should be exercised in using their methodology to estimate tax rates until more conclusive evidence is obtained. Reliable estimates of marginal tax rates might better be obtained by

2 The equilibrium statistic for the non-preferentially taxed sunk-cost seller is \((P_B - P_A)/D = 1/(1 - t_c)\) when transactions costs, \(t_c\), are considered. If, for example, \(t_c = .01\), the equilibrium statistic for the seller is 1.01 rather than 1.0.

3 This perhaps is why E & G’s calculated implied tax rates that decrease substantially with increasing dividend yields are not strongly supported by a recent study using a different methodology (Lewellen et al., 1978).
APPENDIX

Equilibrium Statistics for Security Purchasers

For every seller there must be a buyer. A buyer’s indifference to timing of share acquisition occurs when the after-tax present values of the two decisions, whether at $P_B$ or $P_A$, are equivalent:

$$F_D + \frac{P_A(1 - t_c) - t_{con}[P_A(1 - t_c) - P_B(1 + t_c)]}{\prod_{i=1}^{n} (1 + k_i)} + D(1 - t_c) - P_B(1 + t_c) = F_D + \frac{P_A(1 - t_c) - t_{con}[P_A(1 - t_c) - P_A(1 + t_c)]}{\prod_{i=1}^{n} (1 + k_i)} - P_A(1 + t_c)$$

Equilibrium Condition, equation (A-1)

where $k_i$ is an investor’s after-tax risk adjusted discount rate for period $i$, $P_n$ is the expected share price at $n$, which is the investor’s expected holding time horizon, $F_D$ is the after-tax present value of the dividend to time horizon $n$, and $t_{con}$ is the investor’s expected tax rate for period $n$. Equation (A-1) reduces to

$$\frac{P_B - P_A}{D} = \frac{1 - t_c}{(1 + t_c)(1 - t_c)}$$

Equation (A-2)

where

$$t_c = \frac{t_{con}}{\prod_{i=1}^{n} (1 + k_i)}.$$

Interpretation of the $(P_B - P_A)/D$ statistic is thus impacted by the buyer’s time horizon and anticipated future capital gains tax rates.

If the market is restricted to long-term capital gains sellers and buyers, the ordinary income tax rate, $t_e$, and the capital gains rate, $t_c$, can not be easily derived from the $(P_B - P_A)/D$ statistic if, as can be persuasively argued, the long-term purchaser impacts on the equilibrium statistic. Even with simplifying assumptions of a constant required return, $k$, and $t_{con} = .5t_e$ for an intended long-term buyer. Because of the discounting effect, $t_c < .5t_e$, except perhaps for very short holdings. However, very short holdings are likely to result in ordinary tax on capital gains. Thus, estimates of $t_e$ and $t_c$ in equation (A-2) can not be conveniently derived by assuming $t_c = .5t_e$. Extreme conditions can be specified with $t_e \to 0$ as $n \to \infty$ or $k \to \infty$, and $t_c \to .5t_e$ as $n \to 0$ or $k \to 0$.

Without additional information, derivation of accurate point estimates of $t_c$ from the statistic $(P_B - P_A)/D$ is not possible if the long-term purchaser of a security establishes the equilibrium position. Additionally, if $t_c < .5t_e$, the $E \& G$ estimate of tax rates is overstated. Displacement at the margin among long-term holders would tend toward shorter time horizon holders, since $(P_B - P_A)/D = (1 - t_c)/(1 - t_c)$ as $t_c \to .5t_e$. This expectation is consistent with $E \& G$’s assumption that $t_c \to .5t_e$ and the long-term capital gain seller’s equilibrium condition, equation (2).

If the purchaser’s intended holding period is short, resulting in ordinary income tax on capital gains and $t_c \to t_e$, equation (A-2) reduces to $1/(1 + t_c)$, which is approximately equal to the short-term seller’s equilibrium $1/(1 - t_c)$ if $t_c$ is small. Since this equilibrium condition is $(P_B - P_A)/D = 1$, displacement of long-term holders seems likely.

REFERENCES


PRODUCTIVE EFFICIENCY IN U.S. MANUFACTURING:
A LINEAR PROGRAMMING APPROACH

Henry T. Burley*

I. Introduction

The purpose of this paper is to illustrate the linear programming (LP) activity analysis approach developed from the Farrell (1957) efficiency system, and thus investigate productive efficiency implications in some recently published four factor aggregate U.S. manufacturing data.

This approach, whilst it does not permit determination of statistical significance, has some advantages in that it does not require additive separability of factors in the production function, or the stability of own or cross price elasticities, and avoids some statistical estimation problems arising from multicollinearity in $n$ factor data. The LP approach yields interesting insights resulting from its solution value, optimal basis and slacks. The analysis in the foregoing simple formulation, however, does not allow formal distinction between productivity increase and returns to scale effects; it simply deals with the one category—productive efficiency.

The data arise out of an earlier Berndt and Wood (1975) study which estimates a translog cost function relating total cost as determined by gross output $P$, and

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* La Trobe University.