Summer 2015

COURSE PROPOSAL FOR A MATHEMATICAL MODELING COURSE IN A HIGH SCHOOL CURRICULUM

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By
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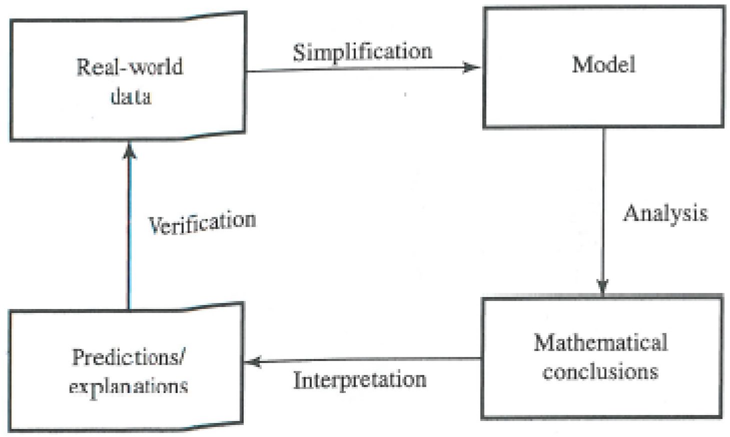
INTRODUCTION

In the winter of 2015, I will be piloting a course on mathematical modeling at Hawken School, an independent high school in Gates Mills, OH. As I develop all elements of this course, such as lesson plans, assessments, and rubrics, I will be mindful of factors such as the newly adopted Common Core mathematics standards, the variety of student backgrounds in such a course, and how various mathematical societies and organizations such as SIAM, MAA, and COMAP can help in implementing it. However, there is one basic question driving my interest in and design of this course: “when am I ever going to need this?” This quote uttered by many high school math students sounds like nails on a chalkboard to teachers who cannot imagine how anyone can take for granted the great beauty in math. While every math student may not appreciate it as an art in its own right, I do believe that every student should be able to appreciate math for its infinite applications. Because of this, I, like many other math teachers, always try to do my best to incorporate many applications of math to a variety of fields in my courses. In doing so, I hope to impress upon them what is probably the most important aspect of mathematics education: the development of critical thinking. However, due to constraints, such as time, adhering to state content standards, etc., such application problems often get pushed to the side in the average math course. This is why the thoughtful addition of a semester long course in mathematical modeling would be an excellent addition to the high school curriculum.

CHAPTER I – BACKGROUND

Any mathematical model that is studied is really just "an idealization of [a] real-world phenomenon" (Giordano 1) and it is important to note that such models can never prove to be a "completely accurate representation," (Giordano 1) but can still provide important insights into the problem at hand. The image below provides a visual approach to the flow of mathematical models (Giordano 1):



As the above diagram may suggest, the mathematical modeling process is not a rigid one; rather, it ebbs and flows, often changing upon different iterations of itself after analyses have been made regarding the various outcomes. Looking ahead, this will undoubtedly prove to be one of the most challenging aspects for students. Unlike a basic algebra problem, there is no neat single answer that students can go back and plug into a single formula to check if they are correct. This, though, may be the very reason why such a course could prove to be such a powerful addition to our curriculum.

It may be helpful to provide some examples of mathematical modeling. Since the definition of a mathematical model is fairly broad as previously stated, the possible examples of a mathematical model problem are endless. Some of the Common Core Standards include, "designing the layout of the stalls in a school fair so as to raise as much money as possible, analyzing stopping distance for a car, estimating how much water and food is needed for emergency relief in a devastated city of three million people, and modeling savings account balance, bacterial colony growth, or investment growth" ("Mathematical Standards"). These examples range over a large spectrum of topics and lend themselves to great cross-curricular studies. Such studies can help "promote pupils' cognitive, personal, and social development in an integrated way" (Savage 6).

Besides providing numerous examples in their standards, the Common Core State Standards Initiative (CCSSI) additionally provides a general modeling process that should be followed when working with such problems. This is a similar process to the modeling cycle that was outlined above by Giordano with the added step of "reporting on the conclusions and the reasoning behind them." ("Mathematical Standards"). This will be something that will be particularly emphasized in the proposed course. Unlike the other high school mathematics standards under the Common Core, *Number and Quantity*, *Algebra*, *Functions*, *Geometry*, and *Statistics & Probability*, *Modeling* is a "Standard for Mathematical Practice." The fact that there is not just a list of ten or so isolated topics related to mathematical modeling, but rather "specific modeling standards appear throughout [the rest of] the high school standards," again reinforces the fact that modeling should be embedded into every part of the high school mathematics curriculum ("Mathematical Standards"). Therefore, a semester-long course in mathematical modeling seems very appropriate.

Over the past several years, the importance of mathematical modeling as an integral part of high school curricula has become a mainstay in several prominent mathematical societies, such as MAA, SIAM, and COMAP. The Mathematical Association of America, MAA, is “the largest professional society that focuses on mathematics accessible at the undergraduate level” (“About MAA”). One of their main goals as a professional organization is in the field of education, where they “support learning in the mathematical sciences by encouraging effective curriculum, teaching, and assessments at all levels” (“About MAA”). This claim is quite apparent with a simple search on their website of “modeling.” Upon such a search, hundreds of books relating to mathematical modeling come up. Books such as *Mathematical Modeling: A Case Studies Approach*, *A Course in Mathematical Modeling*, and *Mathematics in Nature: Modeling Patterns in the Natural World*, and some titles published by the MAA are available for review. Many of these texts could be usefully incorporated into a high school mathematical modeling course.

The Society for Industrial and Applied Mathematics, or SIAM, is another prominent mathematical society that is at the forefront of mathematical modeling. One area that they have been at the epicenter of is helping high school students get involved in mathematical modeling. They have facilitated this through their Moody’s Mega Mathematics Challenge. This is an annual contest open to high school students in which they are given an “open-ended, applied math modeling problem focused on a real-world issue” (http://m3challenge.siam.org/challenge). Not only does SIAM help put on the contest, they also have compiled a free handbook entitled “Math Modeling: Getting Started and Getting Solutions.” While this handbook may be intended for helping students prepare for the Moody's Mega Mathematics Challenge, it would be great as a teaching aide to a high school mathematical modeling course. One of the key strategies for effectively tackling a modeling problem as presented in this handbook is the development of a "mind map," a "tool to visually outline and organize ideas" (Bliss 11). An example of an initial mind map to determine the "best" recycling method is shown below in Figure 1 (Bliss 11):

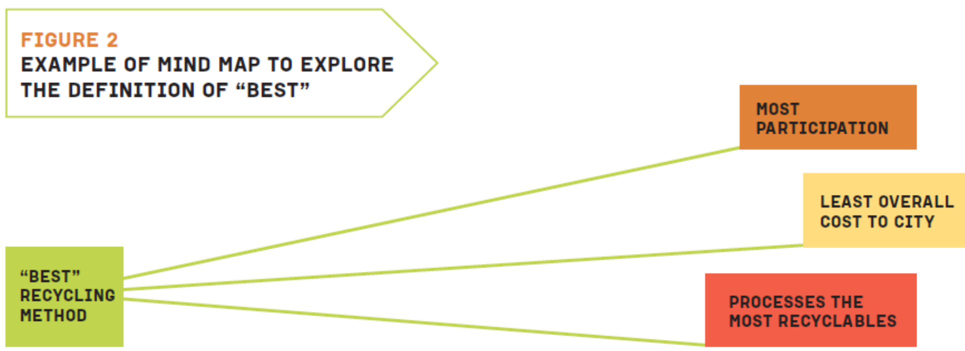


Figure 1

More detailed mind maps can be developed from this initial version, once the "best" method is decided upon. The Moody Handbook gives an example of one in which "best" is assumed to mean the least cost to the city. An example is shown below in Figure 2 (Bliss 12):

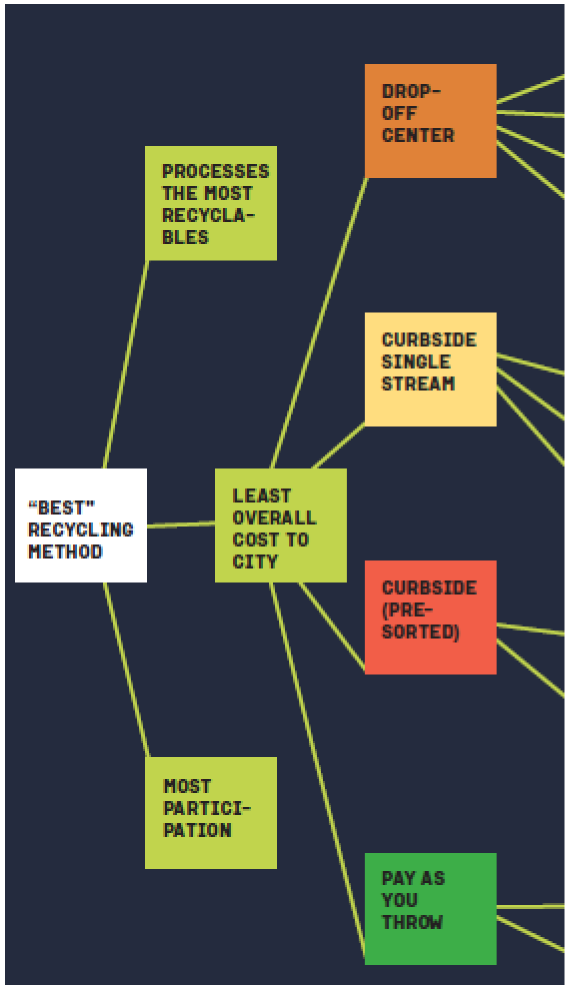


Figure 2

The development of such mind maps as a way for students to brainstorm and think about how the various aspects of their problem are interrelated to one another will be an integral part of this course.

The Consortium for Mathematics and Its Applications, or COMAP, is "an award-winning non-profit organization whose mission is to improve mathematics education for students of all ages" ("COMAP"). One of the more important things they do in the realm of mathematical modeling is the "High School Mathematical Contest in Modeling (HiMCM)." This is an annual contest in modeling, similar to the Moody's Mega

Mathematics Challenge. Their website has an archive of all past year's contests. This collection is an excellent resource to utilize both for teaching specific modeling techniques, as well as assessments for students to demonstrate their knowledge.

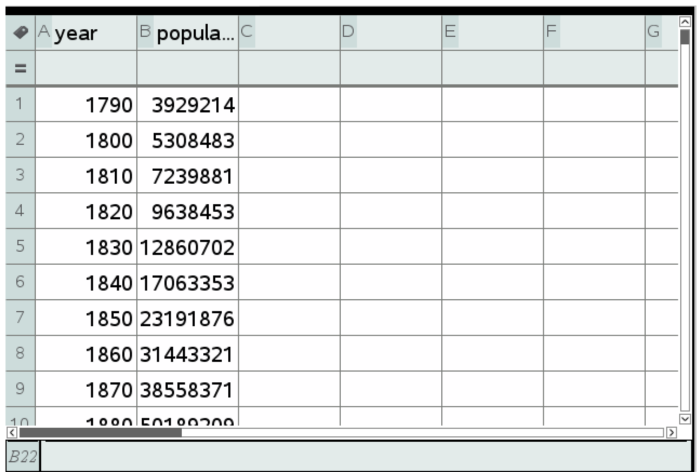
With a boundless number of examples and areas of focus that a modeling course can focus on, the Common Core's emphasis on integrating modeling into the curriculum, and reputable mathematics societies push over the last several years to focus on modeling, the implementation of such a course in a high school curriculum seems natural. What may be slightly less obvious is how to decide on the best implementation of such a course. Important items to consider when implementing a modeling class like this, as well as a proposed plan for the course will be covered in the next chapter.

CHAPTER II – COURSE IMPLEMENTATION

As mentioned earlier, a great thing about mathematical modeling is that it is rooted in so many different branches of mathematics. While the endless possibilities of mathematical application make for great resources of example problems, it does somewhat inhibit one from having this course follow a "natural" planning order. Courses such as Algebra 1, Geometry, and Calculus, tend to have a typical order of topics that most teachers follow. While not every teacher teaches these courses in the same exact way or even the same exact order, there tends to be a natural outline that makes sense to follow logically, and from where lesson planning can then begin. Unfortunately, this is not necessarily the case for a modeling course. Since you could go into any branch of mathematics in a modeling course and spend an infinite amount of time on each of those, a different approach must be taken. I will focus more on bigger problem-solving techniques that tend to come up in many types of modeling problems. Once students are exposed to several different types of techniques, they will hopefully have an "arsenal" from where they can pull out different strategies to help in solving the particular problem at hand. Throughout the course, students will have several opportunities to practice these skills by working on solving modeling problems, many taken from past Mega Moody Challenge and COMAP contests.

Another item to be concerned with when planning a modeling elective course such as this is that there will be differences in the mathematical background between all of the students entering this course. As it stands right now, the modeling course that I will be teaching in the three-week Fall Intensive Semester at Hawken Upper School has sixteen students enrolled in it, coming from several different classes. Since this class will be held in the Fall Intensive, they will already have had a semester of their current year's required math class. Of the sixteen students, juniors and seniors, six will be concurrently enrolled in AP Calculus BC, seven in AP Calculus AB, two in Calculus, and one in Multivariable Calculus. While there will be a wide variety of student levels in this class, it will be nice that in this particular class, every student will have at least a semester's worth of some calculus at their disposal. This would open the door to the study of a lot of different topics that need some calculus to be understood. However, it should be noted that this will not always be the case, and enough flexibility must be left in the overall curriculum to easily change gears from year to year (or semester to semester) depending on the background of the students.

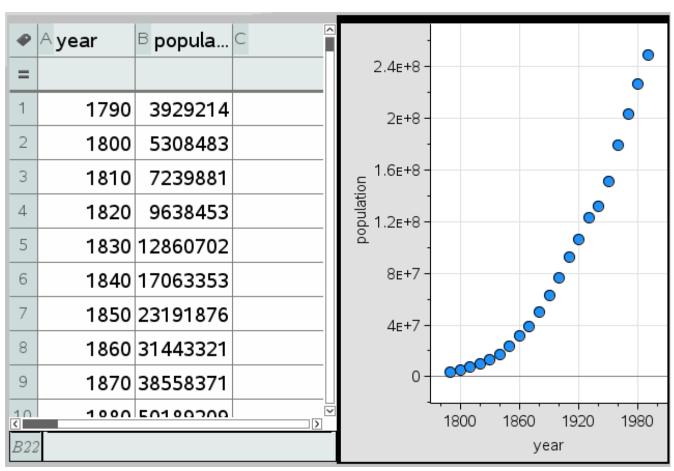
No effective course in mathematical modeling can be implemented without the proper integration of appropriate technology. While there are different types of technologies that could be incorporated into this course, I will focus on two, the TI-Nspire and Microsoft Excel. Since Hawken is a one-to-one school with each student having their own tablet computer already equipped with the TI-Nspire software, the Nspire will be the preferred graphing calculator for student use in this class. However, because this tool does have some limitations, the use of Microsoft Excel will also be encouraged throughout the course. One area that the use of technology is critical for is various regression problems. Given a set of data, the Nspire can quickly determine various regression equations to model a specific phenomenon. For example, if students are investigating population growth, and want to model the population of the United States ("Chart of US Population"), they may choose to input their data into the Nspire, and then look at the various regression options to see which makes the most sense. This can easily be done by first inputting the data into “Lists and Spreadsheets” (see Figure 3 below). This data can then be transformed into a scatter plot by choosing “Quick Graph” (see Figure 4 below). Next, various regression models can be selected, and students are able to compare to original scatter plot and determine which model they believe to be best suited for the particular phenomenon they are studying. Examples of exponential and logistic regression models of the population data are shown in Figure 5 and Figure 6, respectively.



	A year	B popula...	C	D	E	F	G
=							
1	1790	3929214					
2	1800	5308483					
3	1810	7239881					
4	1820	9638453					
5	1830	12860702					
6	1840	17063353					
7	1850	23191876					
8	1860	31443321					
9	1870	38558371					
10	1880	50189209					

B22

Figure 3



	A year	B popula...	C
=			
1	1790	3929214	
2	1800	5308483	
3	1810	7239881	
4	1820	9638453	
5	1830	12860702	
6	1840	17063353	
7	1850	23191876	
8	1860	31443321	
9	1870	38558371	
10	1880	50189209	

B22

Figure 4

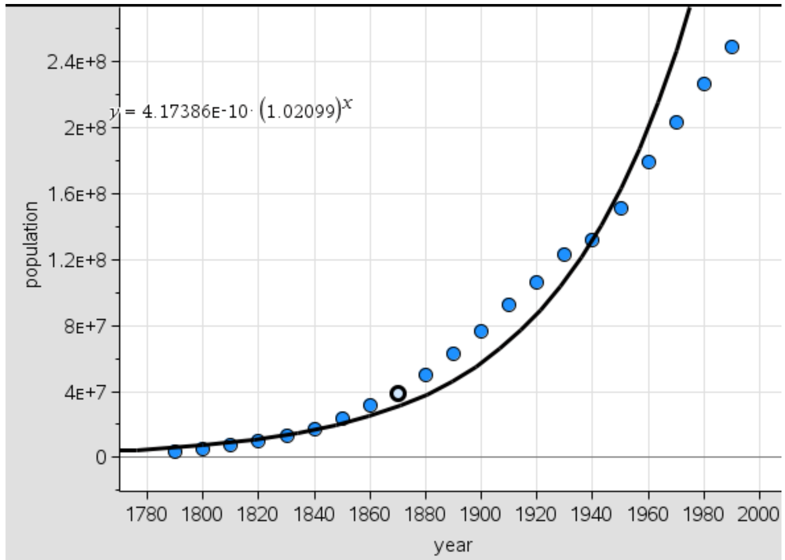


Figure 5

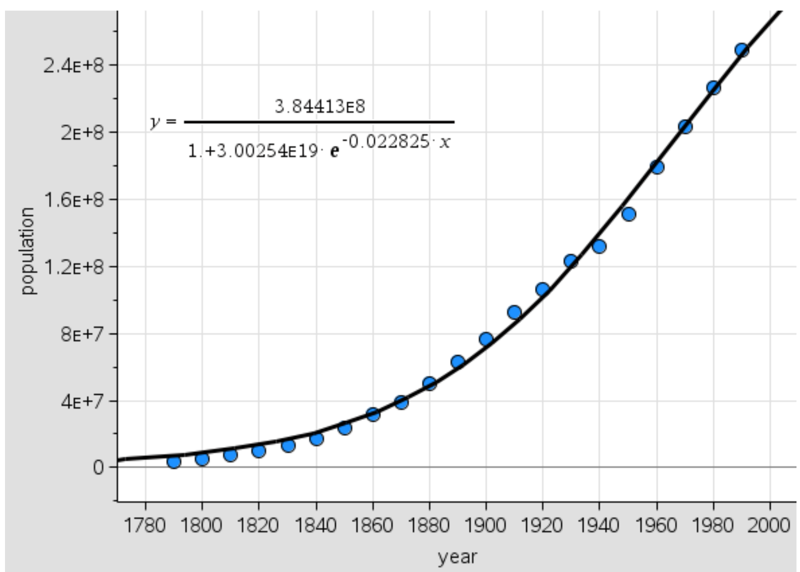


Figure 6

One can hopefully see how the logistic model shown in Figure 6 would be a much better suited fit for a model to describe the data points, as the graph hits almost every one of the original data points. Comparing potential models such as these is very easy to do on the TI-Nspire.

Such a problem can also be solved via Microsoft Excel. The steps are almost identical, but with slightly different syntax. An illustration of what the previous example would look like in Excel is shown in Figure 7 below. An advantage of using Excel is that it is generally quicker to enter the data, create the scatter plot and come up with the regression model, as the syntax with the TI-Nspire software can sometimes be cumbersome, especially for students who have not had much previous experience with it in the past. Another nice thing about Excel, in terms of presentation quality, is the ability to create graph titles, and easily change colors, font, etc. While this may seem a bit superficial, the ease in which Excel can do this may be beneficial for a student forming a final presentation or paper. However, one big disadvantage, which is apparent in this specific population example, is that Excel is limited in the types of regression models it can calculate. There is no option for logistic regression in Excel. The discussion of exponential versus logistic regression when it comes to the example of long-term population growth is an important one, and unfortunately Excel cannot really help facilitate in it.

	Year	US Population
2	1790	3929214
3	1800	5308483
4	1810	7239881
5	1820	9638453
6	1830	12860702
7	1840	17063353
8	1850	23191876
9	1860	31443321
10	1870	38558371
11	1880	50189209
12	1890	62979766
13	1900	76212168
14	1910	92228496
15	1920	106021537
16	1930	123202624
17	1940	132164569
18	1950	151325798
19	1960	179323175
20	1970	203302031
21	1980	226542199
22	1990	248709873

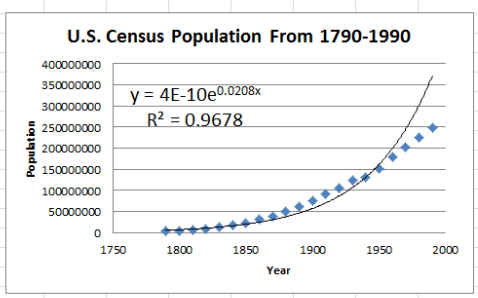


Figure 7

In teaching this course, I would make sure to go over several examples of similar problems using both types of technology, and then leave it to the student to decide which makes the most sense to use in other problems that they will solve.

With a set of core problem-solving topics, an understanding of student backgrounds, and a variety of technologies at my disposal, I was able to then think of student activities, projects, and assessments in order to then develop an actual course plan, included below. It is important to note that this course outline assumes the three-week "Intensive Semester" format at Hawken School. Under this schedule, students attend a single class each day in the three-week period from 8:15 until 3:15. With this schedule comes the added necessity to make sure day-to-day does not become to mundane, making sure to include a variety of topics and activities. While I have tried to account for this in the plan, I am sure that as I actually begin to teach the course, I will amend some of the original plans in order to best facilitate student learning and retention. Although this course plan again assumes Hawken School's unique "Intensive Semester" format, I believe this could be adapted to a regular semester course at other schools with some careful consideration to time and workload. In the next chapter, specific examples of plans I have developed from this initial course plan are included, including sample lesson plans, assessments, grading rubrics, and a course syllabus.

Course Plan

Week #1	Monday	Tuesday	Wednesday	Thursday	Friday
8:15-9:45	Introductions "What If" Activity	Problem From Handbook	Quiz/Test on Concepts so far	Create a "What If"	Field Trip
10:05-11:15	Continue Discussion on Modeling	Work Time	Work Time	Solve together in groups	Field Trip
12:00-1:45	Linear Regression	Technology Options	Present/Critique	Non-Linear Regression	Field Trip
2:05-3:15	Munroe's "What If" Ted Talk	Work Time	Present/Critique	Population Modeling Activity	Field Trip
Week #2	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
8:15-9:45	Talk about last week's presentations	Edit each other's papers	Geometric Similarity	Quiz/Test	Work Time
10:05-11:15	Technical Writing	Probability Distributions	Reading from Dynamics of Dinosaures	COMAP or Moody Problem	Present
12:00-1:45	Examples	Monte Carlo Simulation	Modeling Experiment with Dinosaur Model	Work Time	Present
2:05-3:15	Write up Formal Paper	Stock Market Forecasting	Share/Present	Work Time	Guest Speaker
Week #3	**Monday**	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
8:15-9:45	Test or Quiz	Guest Speaker	Introduce Final Project	Work Time	Work Time
10:05-11:15	TED Talks	Reflection	Work Time	Work Time	Work Time
12:00-1:45	Differential Equations	Review and Practice	Work Time	Work Time	Presentations
2:05-3:15	Practice with Differential Equations	Test	Work Time	Work Time	Presentations

CHAPTER III – SAMPLES

Sample Lesson 1

Title: Introduction to Mathematical Modeling via "What If" Cartoons

Time Required: 3-4 hours spread over multiple days (see tentative schedule in previous section)

Standards Addressed: CCSS.MATH.PRACTICE.MP4 (Model with mathematics)

Materials Needed: Student Laptops, Handouts of "What If" Cartoons, Copy of "What If" book by Randal Munroe

Overview: Scientist and author, Randall Munroe, has compiled hundreds of his "XKCD" cartoons in a book entitled "What If." These cartoons humorously and factually tell the tale of what would happen if bizarre scenarios were to suddenly occur. Cartoons such as "Soulmates" and "Everybody Jump" explain the mathematical and scientific evidence to what would actually happen in such events. These cartoons are a fun and informative way to expose students to the overview of mathematical modeling. After showing a few of these cartoons to students with their explanations, students will receive their own "what if" without a solution. They will be tasked to come up with their best solution to the problem at hand and then share their results. Once shared, we will compare with the actual solution given by Munroe. At a later date students will be tasked with creating their own "what if" cartoon that other classmates will then solve.

Steps:

1) First, I will show on the board an example of an XKCD cartoon.

2) As a class we will have a discussion about possible steps one would take in approaching how to solve the problem as presented in the cartoon, without necessarily going through the entire problem solving process.
3) I will show the class Munroe's solution to the problem. This will lead into a discussion to whether or not the class was on the right track, did we do anything different than Munroe's solution, etc.
4) In groups of four, students will be given a "What If" question that they are tasked to solve. Two questions will be given so that there are two groups with the same question. This will hopefully lead to a nice discussion in comparing and contrasting methods of solutions.
5) Students will have at least an hour to work on their solution to the question as a group. They will be allowed to use any resource available with the exception of Munroe's solution in the "What If" text.
6) Students will present their solutions to the rest of the class.
7) Later in the week, students will be tasked with posing their own "What If" question, coming up with a solution, and an illustrated cartoon.
8) Students will exchange questions with each other and will be tasked to solve each other's questions.
9) Both the group solving the problem and the group that posed the question will present their findings.

Assessment:

Students will be assessed on their individual "What If" cartoons and explanations. A grading rubric for this can be seen next.

Sample Grading Rubric

The following grading rubric will be used to grade students' "What If" cartoons previously mentioned in Sample Lesson #1.

	4 Points	**3 Points**	**2 Points**	**0-1 Points**
Correctness of Solution of First Cartoon	Students give excellent solution explanation that is both factually correct and elegant in nature.	Students give competent solution that addresses all of the issues. Minor errors may appear.	Students give answer that addresses some of the issues but not all. Or, several errors appear in the solution process.	Little to no attempt was made.
Presentation of Solution	Students are articulate and knowledgeable of their solution. At most one mistake in explanation is made.	Students are articulate and knowledgeable of their solution. A few mistakes in their explanation are made.	Students lack articulation and do not appear to be very knowledgeable of either their problem or their solution.	Little to no attempt was made.
Creativity of Cartoon	Students have included a cartoon with at least four different panels. Cartoon is neat and creative.	Students have included a cartoon with three different panels. Cartoon is still neat and creative.	Students have included a cartoon with less than three different panels.	Little to no attempt was made.
Solution of their own Cartoon	Students give excellent solution explanation that is both factually correct and elegant in nature.	Students give competent solution that addresses all of the issues. Minor errors may appear.	Students give answer that addresses some of the issues but not all. Or, several errors appear in the solution process.	Little to no attempt was made.
Solution of Other Cartoon	Students give excellent solution explanation that is both factually correct and elegant in nature.	Students give competent solution that addresses all of the issues. Minor errors may appear.	Students give answer that addresses some of the issues but not all. Or, several errors appear in the solution process.	Little to no attempt was made.

Sample Lesson 2

Title: Voronoi Diagrams and School Districts

Time Required: 3-4 hours

Standards Addressed: CCSS.MATH.PRACTICE.MP4 (Model with mathematics), CCSS.MATH.CONTENT.HSG.MG.A.3 (Apply geometric methods to solve design problems)

Materials: Student laptops

Overview: Applications of geometry come up in several different mathematical modeling problems, including several of the problems presented in past COMAP and Mega Moody Challenge contests. Therefore, I think it will be beneficial for students to review some important geometry skills and then think about how they could apply them into various real-world situations. After spending some time reviewing key properties about triangles and their centers (circumcenter, incenter, etc.), I will introduce the concept of Voronoi diagrams. Students will then complete a step-by-step walkthrough where they utilize the idea of Voronoi diagrams to determine the ideal placement of school districts in their area. They will then research the actual boundary lines of those school districts and compare. Finally, as a class we will look at how Voronoi diagrams helped in the understanding of what happened during a cholera outbreak in London in the mid-1800s.

Steps:

1) First, we will review the definitions of perpendicular bisectors, medians, altitudes, angle bisectors, orthocenter, circumcenter and incenter.

2) Students will complete step-by-step walkthrough on Voronoi diagrams and school districts (see appendix)
3) After students finish, we will have a discussion about their findings. We will talk about how and why the actual school district boundaries differ from the "ideal" boundaries.
4) I will introduce the story of the cholera outbreak in London in 1853 and discuss how Voronoi diagrams helped isolate the problem (https://plus.maths.org/content/uncovering-cause-cholera)
5) We will discuss what other types of modeling problems might utilize the idea of Voronoi diagrams or other triangle centers.

Assessment: Students will be assessed on their completion of the Voronoi diagram activity (see Appendix A).

Sample Lesson 3

Title: Geometric Similarity and Dinosaurs

Time Required: 2 hours (over two days)

Standards Addressed: CCSS.MATH.PRACTICE.MP4 (Model with mathematics), CCSS.MATH.CONTENT.HSG.MG.A.3 (Apply geometric methods to solve design problems)

Materials: Reading excerpts from *Dynamics of Dinosaurs and Other Extinct Giants*; cardstock; straightedges; string; paperclips; student laptops

Overview: Modeling real-life objects to well-known basic geometric shapes is an important skill, and one that appears in some of the past modeling problems from COMAP and the Mega Moody Challenge. This lesson will help relate finding the centroid of basic geometric shapes to finding the center of mass of more irregular shapes – like dinosaurs!

Steps:

1) On the first day that this lesson is introduced, we will review how to find the centroid of a triangle. Students will do this both physically (with cardstock, straightedge, and compass) as well as virtually (on Geogebra).
2) Next, students will be given the same task as the previous step, but for a quadrilateral.

3) The final thing that students will do on the first day is do the same task but with an irregular shape. The will have access to the cardstock, paperclips, string, and Geogebra. No specific instructions will be given on how to do anything, however.
4) Students will get into small groups and discuss their strategies for Steps 2 and 3. We will come together as a class to discuss various strategies.
5) For homework, students will read various excerpts from *Dynamics of Dinosaurs and Other Extinct Giants*.
6) On the second day of this lesson, we will discuss the reading as a class.
7) Next, students will be given a copy of models of dinosaurs from different viewpoints. Their task will be to locate the center of mass of these dinosaurs.
    a. Students will be given a few minutes to work on this individually
    b. Then students will get in small groups to compare and come up with a final solution and explanation as to why they decided on their center of mass.
8) Students will then be given an opportunity to research online to see if they can find out where the actual center of mass for these dinosaurs was suspected to be. They will then compare to their results and see how close they were.
9) We will discuss as a class our findings, comparing different strategies of finding the center of masses, and then talk about other relatable applications.

Assessment: Students will be given a small quiz later in the week where they have to find the center of mass of both a quadrilateral and an irregular shape. Students will be expected to not only find the center of mass, but also explain their thought process and steps.

Sample Assessment 1

As stated earlier, the idea of "mind maps" will be something that is stressed in this class. Early in the first week of the course, I will assess students on their ability to create a mind map for a new modeling problem they have yet to see. They will not be asked to try to solve the entire problem; rather, they will just be tasked with creating a mind map, outlining all of the possible aspects of the problem they should be familiar with, if they were to actually solve the problem. They will also be asked to determine the variables (both independent and dependent) and any assumptions that they will make. An example of what this question might look like on an in-class quiz is included in Appendix B.

Sample Assessment 2

As a culminating project to the course, students will be tasked with coming up with their own mathematical problem that they will then take action on and solve. The format of this assessment will be similar to COMAP's High School Mathematical Contest in Modeling (HiMCM), with the exception that they will be the ones coming up with the problem. This project will be introduced to the class in the early part of the third week, giving them sufficient time to think about what problem they want to tackle. They will have the last three days of the course to work on solving the problem, write up their solution in a technical paper, and then present their findings to the rest of the class.

Sample Course Syllabus

Course: Mathematical Modeling (Intensive)

Location: Ireland Room 1

Teacher: Mr. Marlowe

Email: tmarl@hawken.edu

Phone: 440-423-4446 x 419

Text: No textbook required; various readings will be provided for students

Grade:

- **Final Project:** 50%
- **In-Class Quizzes** 20%
- **In-Class Projects:** 20%
- **Participation:** 10%

Material Covered – A tentative timeline for this course is as follows:

Week 1: Introduction to Mathematical Modeling; Mind Maps; Technology Overview; Linear Regression; Non-Linear Regression

Week 2: Technical Writing; Probability Distributions; Monte Carlo Simulations; Geometric Similarity

Week 3: Differential Equations; Final Projects

Overview: This course in mathematical modeling will challenge students to think about math in a different light than previous math courses. The focus of this course will be on developing problem-solving strategy skills to solve a variety of real-world applications stemming from a variety of mathematical branches. Other items of focus include group collaboration and the development of technical writing skills. This course will culminate in a final project where students will pose their own problem that they will then solve.

Assessments: Assessments in this course will include smaller in-class quizzes testing students on various skills learned in class, as well as larger projects which will be completed both inside and outside of class. As mentioned previously, this course will culminate in a final project where students will pose their own problem that they will then solve.

Homework: Students should expect to be working on this course outside of normal class time on a nightly basis. While typical math "problem sets" will not likely be assigned, students are expected to keep up on readings, work on long-term projects and study for in-class quizzes.

Materials Needed: *Pencils and Erasers*
Laptop/Tablet
Folder/Binder
Geometry Tools: Compass, Protractor, Ruler

Communication: Students should feel free to contact me with questions about homework assignments. I will usually check my e-mail and respond once per night. Also, you and your parents can contact me on my school phone, listed at the top of this syllabus, which will be checked twice during the school day.

Collaboration: Showing your work is truly important in all math classes, but doing your own work is equally important. You will be taking each assessment without the help of classmates or solution manuals so the best way to prepare for these opportunities is to do the work, ask questions when you are unclear and conceptualize how the topics are applied.

Simply put, you may NOT just copy someone else's work, in any circumstance. That includes, copying with their permission, and having someone on a computer help site do the work for you. I encourage you to use the many resources you have but, in a fashion that will help you achieve the ultimate goal of understanding the material.

Make-Up Policy: It is the responsibility of the student to acquire all assignments and material covered during an excused absence, and to submit all assignments due during the absence upon his/her return. Students who miss school for an unplanned, excused absence of five days or fewer are entitled to 1.5 days per day of absence, rounded up to the nearest whole day, in which to make-up all missed work and tests and remain current with the class.

Hawken Upper School Integrity Code:
As a member of the Hawken community,
I am a person of integrity striving to be my better self.
My words and actions reflect my belief injustice, compassion and fair play.
I respect the rights, work, ideas and dignity of all.

CHAPTER IV – CONCLUSION

Through my research and development of this course plan, I have reaffirmed my belief that the implementation of a mathematical modeling course in a high school curriculum is of the utmost importance. Such a class will not only expose students to a plethora of different math concepts that they might not otherwise have the opportunity to see anywhere else, but also have them practice one of the most important skills that students should be practicing in high school, the art of problem solving. While the technical skills introduced in algebra, geometry, trigonometry, and calculus are extremely important, if students cannot figure out how to utilize them effectively and efficiently in order to solve a new problem they have never seen before, they do not serve a whole lot of purpose on their own.

While I am sure I will experience a lot of work and some frustration in implementing this course for the first time this winter, I expect there to be multiple rewards as well. I anticipate seeing a growth in students' critical thinking skills and a better understanding of how important math truly is. I hope to see what works and what does not work in a course like this in the unique academic setting that Hawken has to offer. Afterwards, I hope to reflect and adapt so that I can improve the course for future iterations. Additionally, if it is successful, I hope to share my findings with teachers at other schools so they may also think about developing similar courses at their own schools.

References

"About MAA." *MAA*. Web. 4 June 2015.

Bliss, K.M., K.R. Fowler, and B.J. Galluzo. Math Modeling: Getting Started & Getting Solutions. Philadelphia: Society for Industrial and Applied Mathematics, 2014.

"The Challenge." *The Challenge*. Web. 4 June 2015.

"Chart of US Population, 1790-2000." *Chart of US Population*, 1790-2000. Web. 17 June 2015.

"COMAP." *The Consortium for Mathematics and Its Applications*, 2015. Web. 1 June 2015.

Giordano, Frank R. *A First Course in Mathematical Modeling*. 4th ed. Belmont, CA: Brooks/Cole, Cengage Learning, 2009.

Gould, Heather, Diane R. Murray, and Andrew Sanfratello, eds. *Mathematical Modeling Handbook.* Bedford: COMAP, 2012.

"Mathematical Standards." Common Core State Standards Initiative, 2015. Web. 26 June 2015.

Savage, Jonathan. *Cross-curricular Teaching and Learning in the Secondary School*. Milton Park, Abingdon, Oxon: Routledge, 2011.

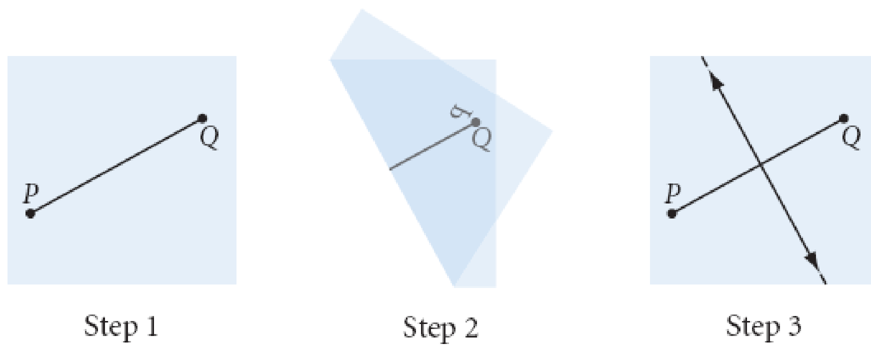
"Uncovering the Cause of Cholera." *Uncovering the Cause of Cholera*. Web. 26 June 2015.

Appendix A

Voronoi Diagram & School District Instructions:

Include all of your answers in this OneNote page. Please write or type in a different color so I can easily see your answers.

1. Draw two points P and Q anywhere on a sheet of paper, connect them with a segment $\overline{PQ}$. Fold the paper to make the end points match. Unfold the paper.



2. Pick a random point on one side of the crease. Which point is it closer to, P or Q?

   Pick a random point on the other side of the crease. Which point is it closer to, P or Q?

   Pick a random point on the crease. Which point is it closer to, P or Q?

3. Describe the relationship between $\overline{PQ}$ and the line formed by the crease.

4. The same concept that the previous paper folding activity demonstrated applies in many real-world situations. For example, if there are two cell phone towers near you, your phone will be connecting to the one that is the nearest. In Geogebra, draw two points (A and B) representing two phone towers. Draw a line that separates the area closest to one tower from the area closest to the other one. Snip a screenshot and paste the sketch below.
   Note: there is a tool in Geogebra that allows you to do it precisely. Find it!

   Using a highlighter, highlight the area that is closest to Cell Phone Tower A with yellow. Highlight the area that is closest to Cell phone Tower B with green.

5. Repeat the same process with three cell phone towers represented by three *non-collinear* points (A, B, and C). Paste the sketch below and shade the areas corresponding to each of the three towers. This type of diagram is called **Voronoi Diagram** (after Georgy Voronoi, 1868-1908)

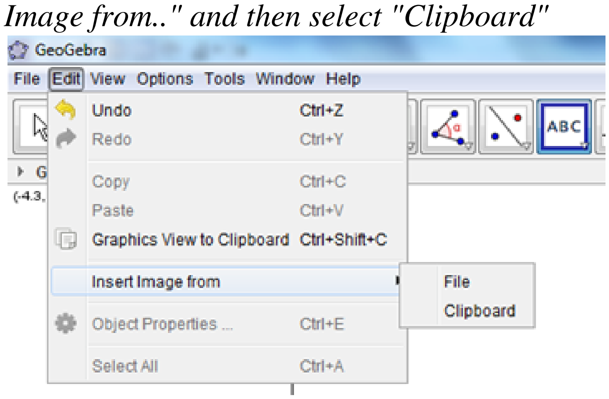
a. Using a highlighter, highlight the area that is closest to Cell Phone Tower A with yellow. Highlight the area that is closest to Cell Phone Tower B with green. Highlight the area that is closest to Cell Phone Tower C with pink. When you are done there should be no white space, and there should be no overlapping colors.

6. The same concept can be also applied in dividing a residential area into school districts. Explain why a Voronoi diagram may help in dividing school districts.

7. Find a map that includes the area in which you live and mark on it the location of all local public high schools . The best way to do this is to search for your address in Google Maps (maps.google.com) and then search for "public high schools". Zoom out until you have at least **three**. Note that some schools that will come up are not public or not high schools. **Make sure the three schools you have on your map are three public high schools**. Otherwise you won't be able to finish the rest of the assignment correctly.

8. Import the picture into Geogebra and draw the Voronoi Diagram for these high schools on the map. Paste the drawing below. The following directions tell you how to import a picture into GeoGebra:

Note on importing a picture into Geogebra:
1. Clip the picture with the Snipping Tool
2. Copy the picture
3. Insert the picture into Geogebra by selecting the "Edit" menu and then "Insert Image from.." and then select "Clipboard"



a. *Resize and move the picture to fit your needs.* ***Right-click on the picture and select "Fix Object"*** *This will prevent it from moving by accident.*

b. Include a snip of your Voronoi Diagram for the school districts below, and shade the regions like you did in Step 5a

9. Look up the actual boundaries of the school districts. A good resource is http://www.greatschools.org/school-district-boundaries-map/

10. Copy them below (you can copy and paste the three districts individually, or, you may try to assemble the three district maps into one map and trace the boundaries that are given from that website.)

11. Write a paragraph describing the differences between the Voronoi Diagram and the actual boundaries. What are the possible reasons for the differences?

Appendix B

Mind-Map Quiz

Name ____ Date ____________

Given the following scenario taken from the 1999 COMAP HiMCM contest, develop a "mind-map," outlining the important aspects that you would address if solving the problem. You do not and should not actually solve the problem. Feel free to use all of the space below and/or the backside of this paper.

Then, below the mind-map, identify all the variables of the problem (both independent and dependent) as well as the assumptions you will make.

"Major thoroughfares in big cities are usually highly congested. Traffic lights are used to allow cars to cross the highway or to make turns onto side streets. During commuting hours, when the traffic is much heavier than on any cross street, it is desirable to keep traffic flowing as smoothly as possible. Consider a two-mile stretch of a major thoroughfare with cross streets every city block. Build a mathematical model that satisfies both the commuters on the thoroughfare as well as those on the cross streets trying to enter the thoroughfare as a function of the traffic lights. Assume there is a light at every intersection along your two-mile stretch.

First, you may assume the city blocks are of constant length. You may then wish to generalize to blocks of variable length."
(http://www.comap.com/highschool/contests/himcm/1999problem.html)

The essay of Thomas P. Marlowe is hereby accepted:

Advisor – Brendan Foreman

Date

I certify that this is the original document

Author – Thomas P. Marlowe

Date