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Analysis of Music Note Patterns Via Markov Chains

Ala'a Wadi
John Carroll University, awadi13@jcu.edu

ANALYSIS OF MUSIC NOTE PATTERNS VIA MARKOV CHAINS

by

ALA’A WADI

John Carroll University

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This Senior Honors Project, **ANALYSIS OF MUSIC NOTE PATTERNS VIA MARKOV CHAINS**

has been approved.

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honors program reader date

ABSTRACT

This is a presentation of a novel method for measuring the distance between two seemingly analogous fragments of music, as deemed by human perception. This is an approach entirely based on coarse-grained and primitive representations of the order of the notes that make up the songs. Through the use of a simplified Markov chain analysis, transition matrices are derived for each music piece and compared through linear algebraic techniques.

We begin by applying the method to two distinctive, intrinsic, and familiar compositions, “Row Your Boat” and “Happy Birthday”. We will advance to more complicated compositions that have garnered attention for being “too similar,” the first set being Vanilla Ice’s “Ice Ice Baby” and Queen and David Bowie’s “Under Pressure,” and the second, a more recent set of songs, “When Love Takes Over” by Kelly Rowland and David Guetta versus “Clocks” by Coldplay. Via the use of a Markov chain analysis and matrix algebra, we discover hypothesized results of small-distanced values and unforeseen values that were initially thought to be small but actually indicate large distances between music compositions. Since notes are the foundations to music, these results relate to the identities of separate music compositions by distinctive artists in disparate genres.

I. INTRODUCTION

Vanilla Ice instantly became a household name when his popular hit “Ice Ice Baby” pervaded teenager’s boom boxes and Sony Walkmans nationwide. He was rap’s new “it boy,” known for his unique dance moves and the signature blond streak in his hair. But with fame came criticism, and Vanilla Ice was publicly reprimanded for his sampling of Queen and David Bowie’s “Under Pressure”. When initially confronted in an interview about the rumor, Vanilla

Ice claimed his song "did not sound anything like 'Under Pressure'" ("Vanilla Ice Interview"). In the same interview, Vanilla Ice admits to sampling "Under Pressure" and only adding in a small change to the beat. Nonetheless, the case never went to court, for that it was clear that the bass line of his number one song was taken directly from Queen and Bowie's song. The case was settled out of court, likely resulting in a large sum of money handed over from Vanilla Ice to Queen and Bowie for the copyright infringement ("Famous Copyright Infringement"). Vanilla Ice's case is not unique; many copyright infringement cases in the music industry have seen the inside of a courthouse for as long as the legal system has existed. Due to the difficulty of pinpointing copyright infringement, there seems to be a need for an efficient method that distinguishes a difference (or lack thereof) between two pieces of music. This communication, through the use of Markov chains and transition matrices, provides an innovative, although primitive, method as evidence beyond the human ear that can be considered useful in detecting similarity between two compositions.

Mathematics is the main infrastructure for this method; pattern-extracting structures called Markov chains acting as the foundation and the song composition's note transitions and the transition matrix acting as the walls and floors of the building. *Markov chains* are defined as sequences of random variables with the property that given the present state, the future and past states are independent. These chains utilize *transitions* between notes in order to compare a set of songs. Transitions are changes from a previous state to a current state, and these transitions are depicted in a tool called a *transition matrix.* This type of matrix describes the transitions of a Markov chain, with each entry representing a probability. To further deconstruct the explanation of this type of matrix, each row and column represents a different note on the music scale, as shown in the basic example below:

$$\begin{array}{c} \\ A \\ B \\ C \end{array}\begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.5 & 0.4 & 0.1 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} \end{array}$$

For sake of simplicity, we are considering only three notes, A, B, C. In the music composition that this transition matrix corresponds to, the probability that the note transition $A \to A$ occurs is 20%, $B \to A$ is 50%, and $C \to B$ is 20%, etc. Thus every music composition that will be studied will correspond to a transition matrix of this type.

It is also necessary to review a few music fundamentals. In traditional music – down to its most basic form – we have the notes A, B, C, D, E, F, and G. As we will discuss later on, many other factors affect how music sounds. However, the method used in this literature is based on the order of the notes from a piano scale, which can be seen in the figure below:

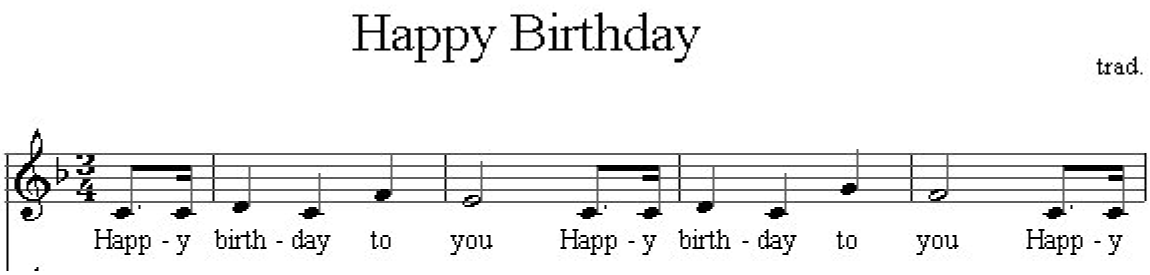
Credit: Andrew A.

By using the scale from above, we perform the tedious practice of writing out the letters of the notes (as shown in the figure) and tallying up the note transitions in each piece (this is further explained in the “Methods” portion of the paper). Thus begins the process of “quantifying music” – counting the number of basic note transitions and transforming them into probabilities to be inputted into a transition matrix. Next, we discuss two methods involving matrix algebra in hopes of measuring a “distance” between two matrices representing two song compositions.

II. METHODS

Creating a Transition Matrix

To provide a simple example of the process used in determining similarity between two songs, we begin with the most famous song in the English language – "Happy Birthday" ("Fun Facts"). For demonstrational purposes, the first stanza of the song is featured below:



[1]

After recording the notes for the first stanza of this sample, we have the sequence "*CCDCFECCDCGFCC*". In this sequence, the note transition $C \to C$ occurs three times out of thirteen total note transitions. Similarly, the note transition $C \to D$ occurs twice. The following is a list of note transitions from the given sequence:

$$
\begin{gathered}
C \to C : 3 \\
C \to D : 2 \\
D \to C : 2 \\
C \to F : 1 \\
F \to E : 1 \\
E \to C : 1 \\
C \to G : 1 \\
G \to F : 1 \\
F \to C : 1
\end{gathered}
$$

To begin creating a transition matrix, we first created a matrix with entries that denote the amount of times a specific note transitions to another note:

[1] Appendix A

$$FirstStanzaOfHappyBirthday := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 2 & 0 & 1 & 1 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

To reiterate, the columns are ordered A through G, as well as the rows, so the (3,3)-entry corresponds to the note transition $C \rightarrow C$, and the (6,5)-entry corresponds to the note transition $F \rightarrow E$. Since this is technically not a transition matrix (a transition matrix's rows should add to one), we divided each value in the above matrix by the sum of its row's values, resulting in the following transition matrix:

$$FirstStanzaOfHappyBirthday := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{3}{7} & \frac{2}{7} & 0 & \frac{1}{7} & \frac{1}{7} \\ 0 & 0 & \frac{2}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{1} & 0 & \frac{1}{1} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{1} & 0 \end{bmatrix}$$

Every row of this final matrix contains values that add to one, as an appropriate transition matrix should.

III. ALGORITHMS/RESULTS

Finding Distance – First Algorithm

Given two transition matrices, this algorithm involves finding the difference between corresponding entries in each matrix and considers the “distance” to be the maximum difference of all of the entries. In other words, we subtract one matrix from the other, and then take the absolute value of the resulting “difference” matrix. It should be noted that all matrix operations were completed using Maplesoft’s Maple 16. We extracted the largest-valued entry in the “difference” matrix and considered this the distance between the two songs. Reason for finding the largest-valued entry is because this value indicates the biggest possible difference between the two matrices. We should note that as it is possible to try to find some mean, median, or weighted difference of the entries rather than taking the maximum value to be the distance, this method was constructed to provide a “simple and straightforward” algorithm to compare to a second, more complicated, algorithm that will be introduced later. Thus if two songs have a small “maximum value”, then the songs indicate resemblance. At this point, it was imperative to know what value would be considered a large distance (implies little similarity between songs) in order to know what value would be considered a small distance (and thus would imply considerable similarity between songs). To make the most of the time that was spent practicing the note transitions in “Happy Birthday”, it was decided that this composition will be compared to a song that we know sounds substantially different but just as manageable – the American childhood classic “Row Your Boat”[2]. Through using this algorithm, the resulting value was 0.8. Hence this is what is considered to be a “big” value since the two compared songs sound nothing

[2] Appendix B

alike to the human ear. This is helpful because it provides us with a reference point for our distances from other sets of songs.

Next we intend on utilizing the same algorithm on the set of songs in question – Vanilla Ice's "Ice Ice Baby"[3] and Queen and David Bowie's "Under Pressure"[4] (we will refer this set as Set 2). From first glimpse at the note sequences of the two songs, it is clear that the songs are almost identical. Take for example a relentlessly repeated segment of the first stanza: "Ice Ice Baby"'s note sequence is "*DDDDDDAD*" whereas "Under Pressure"'s note sequence is "*DDDDDDA*". Luckily, the algorithm seems to agree since the result is 0.012821, which we assume seems to be considered a small value in comparison to our reference point of 0.8. The small value implies a minute difference between the notes of the songs and thus we can consider the songs dangerously similar. By expanding this research, it is possible to find a value that would be considered the boundary line/threshold that separates songs that are too similar from songs that are "different enough". A value such as 0.012821 would be something interesting to examine further because it would seem as if a value this small would be considered one that has "crossed the line" or surpassed a given threshold (as in statistics with the threshold often being $\alpha = 0.05$ and finding a p-value of 0.01). It is easy to deduce that this simple algorithm is not the only way we can discover similarity between two song compositions, as we will witness with our second algorithm.

Finding Distance – Second Algorithm

With credit given to Steven J. Leon and his book "Linear Algebra with Applications", matrix norms were decided to be an adequate way to measure distance between two matrices. Thus arises our second algorithm – given any two matrices *A* and *B*, the objective is to find the

[3] Appendix C
[4] Appendix D

product $\left(B-A\right)^{T}\left(B-A\right)$, then find the trace of the resulting matrix, and finally, take the square root which will result in the matrix norm, or the "distance" from *A* to *B* (Leon 403-409).

For clarification, take the matrices *A* and *B:*

$$A := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad B := \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5.01 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

It is no coincidence that these two matrices only differ by 0.01 in the (2,2)-entry. We want to examine what we know are two almost exact matrices and find the "distance" between them. The distance should be small, and this algorithm turns out to accurately reflect our hypothesis. If we find the product $\left(B-A\right)^{T}\left(B-A\right)$, we have

$$\begin{bmatrix} 0. & 0.01000000000 & 0. \\ 0. & 0.03000000000 & 0. \\ 0. & -0.01000000000 & 0. \end{bmatrix}.$$

The trace of this product is 0.03, and thus our "distance" is $\sqrt{0.03} = 0.173205$. As we expected, the "distance" of this pair of matrices is what we consider to be small. Since every entry of the difference matrix is taken into consideration in this algorithm, it seems to be a more wholesome measure than simply finding the maximum value of the difference matrix like what the first algorithm does.

Applying this algorithm to the set "Happy Birthday" and "Row Your Boat", the output is a value of 1.92732, which we will use as a reference point for the second set of songs. Applying this algorithm to the second set yields a value of 0.018131, which in comparison to 1.92732, is a very small value, ultimately suggesting prominent similarity between the two songs. To present a more condensed form of our findings thus far, we offer the information in the following table:

	Reference Point	Distance for Set 2
Algorithm 1	0.8	0.012821
Algorithm 2	1.92732	0.018131

One should note that through using two different algorithms, similar results were found nonetheless in the analysis of Vanilla Ice's hit single and Bowie's song.

Introducing a Third Set

After careful consideration and for the purpose of "telling a story" through this analysis, it was decided that as long as time permitted, that a third set should be introduced and that this third set should be a representation of today's music. From "Happy Birthday" that dates back to the mid-19th century to 1981's "Under Pressure", the story continues with the 21st century's "When Love Takes Over"[5] by David Guetta & Kelly Rowland and "Clocks"[6] by Coldplay ("Fun Facts"). To the human ear, the introductions of the two songs of the third set sound profoundly similar (Wagner). This congruency is the determination for applying the algorithms to the set – and so we began recording note transitions. To our dismay, the results were not what we expected, with a value of 0.66667 from the first algorithm and a value of 1.9560 from the second algorithm. We add these to our existing table:

	Reference Point	Distance for Set 2	Distance for Set 3
Algorithm 1	0.8	0.012821	0.66667
Algorithm 2	1.92732	0.018131	1.9560

This is a problem because our distances from a set of songs that we hypothesized would yield a small value actually yielded values that were close to or larger than those of a set of songs that sound nothing alike. The issue with large values is that we are unable to conclude that the songs are anything alike using mathematical evidence. Consequently, we began to explore other factors

[5] Appendix E
[6] Appendix F

(besides order of notes) that might be affecting our results. After analyzing the sheet music of all of the sets, we found that every set of songs had songs that started off with the same note, except the third set. To counteract the effect of different starting notes, we decided to shift every note of one of the songs so that the songs will start off on the same note. Rowland's song starts with a C whereas Coldplay's song starts off with an E, indicating a whole note difference. After shifting every note in "Clocks" down a whole note on the scale, note transitions were cataloged and put into a revised transition matrix. Applying the first algorithm produced a distance of 0.80952 and from the second was a value of 1.7396, all documented in the following updated table:

	Reference Point	Distance for Set 2	Distance for Set 3	Distance for Shifted Set 3
Algorithm 1	0.8	0.012821	0.66667	0.80952
Algorithm 2	1.92732	0.018131	1.9560	1.7396

The two new values should most definitely come as a surprise, considering that using the first algorithm yields a larger value than that of the distance for the un-shifted matrix meanwhile the second algorithm yields 1.7396 as opposed to the old value of 1.9560 (a value smaller than that of the third set's, but still considered large). Although the third set may sound genuinely similar to the human ear, our algorithms fail to show any indication of similarity based on the distances. After examining the music compositions of the shifted "Clocks" matrix and its partner, we discovered that these large distances are due to the fact that the compositions do not have the same notes, but they have the same rhythm – which is why they sound alike. Our algorithms, on the other hand, only focus on the order of the notes and do not take into account the rhythms of the songs. Rhythm accounts for a significant part of why songs sound like they do – they are the sounds and silences that form a pattern of sounds that are repeated to create the rhythm. It has a beat that distinguishes strong, long, short, and soft parts of the song. Since our algorithm only addresses only the notes, and the notes of the two songs are different, then the algorithm thinks they are different and concludes that the songs are different.

IV. CONCLUSION

We might have come up a little short with our initial hypothesis, but this communication nonetheless provides an inventive approach that encourages mathematicians and computer scientists to create algorithms and programs that would help with solving the real-world problem of copyright infringement cases in the music industry. Likely, there are reasons for why our results did not coincide with our early theories. This analysis has many obvious limits and several areas in which research can be conducted in order to further explore our objective of determining distances between song compositions. One of the major influences on the outcomes of the examination is solely focusing on the order of the notes and disregarding other music elements that make a song sound like it does. To hone in on the example of the third set, concentrating merely on the order of the notes did not permit us to confirm similarity between the two songs. Perhaps if we had found a way to enumerate factors such as rhythm, tempo, or duration, we could have concluded with more promising results. Additional factors that may have had an effect on this research include: analyzing portions of songs rather than whole songs, neglecting the existence of melody, pitch, keys, accidentals, the fact that different instruments use different scales, and various others. A generous amount of time and resources would be necessary to observe only a few of these additional factors, let alone all of the ingredients that compose music. Another key to consider is the possibility of using other algorithms – or the sophistication of the basic algorithms proposed in this research – that utilize some of these additional factors. Then maybe it would be possible to find more accurate measures of distances between song compositions. With the proper resources and determination, this research can hopefully be extended to help dispute plagiarism in the music industry. Maybe then it would be possible to further explain to Vanilla Ice exactly how *little* of a difference his extra note made in the beat of his song and to deter contemporary music artists like David Guetta and Kelly Rowland from following in Vanilla Ice's footsteps if they want to stay out of the courtroom.

V. APPENDICES

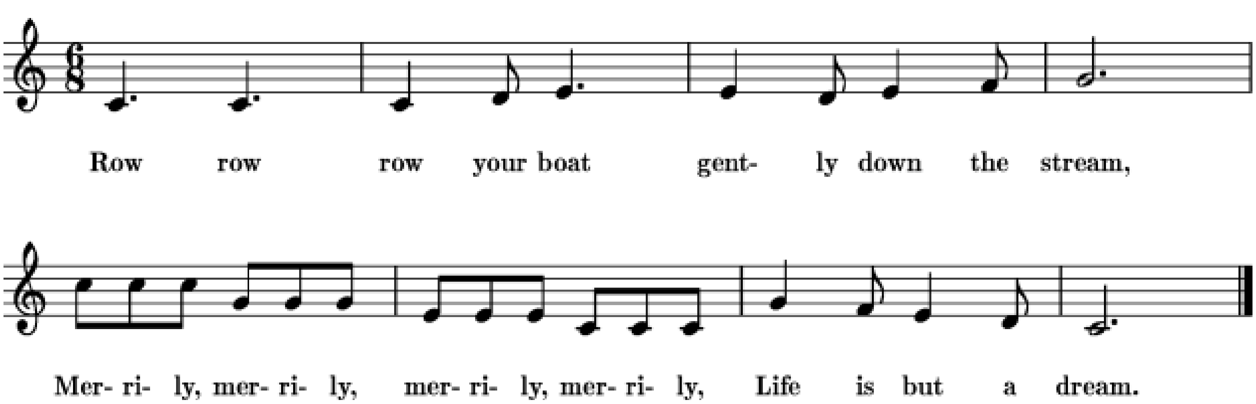
Appendix A

Free sheetmusic from www.8notes.com

Happy Birthday

trad.

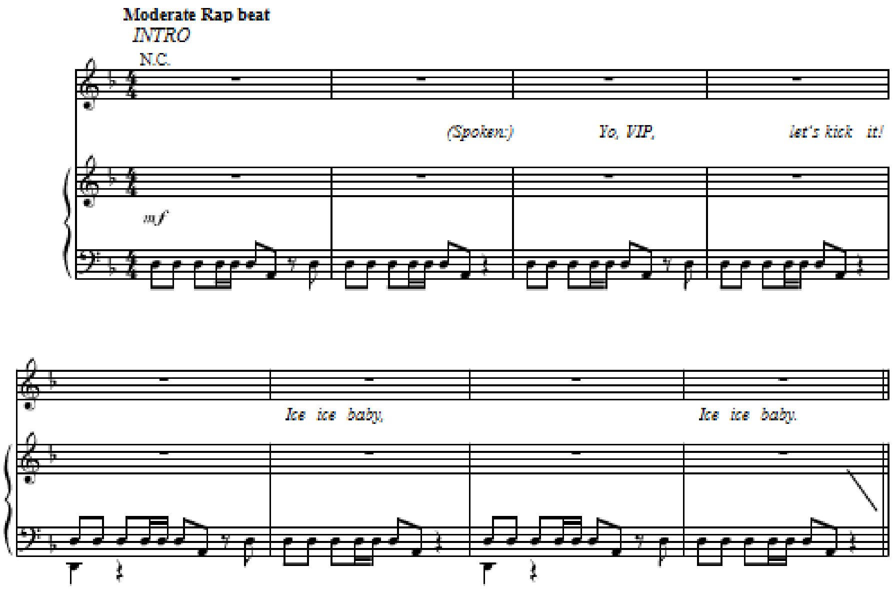
Appendix B



Appendix C

ICE ICE BABY

Words and Music by VANILLA ICE, EARTHQUAKE, M. SMOOTH, BRIAN MAY, FREDDIE MERCURY, ROGER TAYLOR, JOHN DEACON and DAVID BOWIE



Appendix D

Under Pressure

Words & Music by David Bowie, Freddie Mercury, Roger Taylor, John Deacon & Brian May

Appendix E


When Love Takes Over
Words & Musicby KellyRowland,David Guetta,
FredericRiesterer, MiriamNervo & OliviaNervo
Originalkey F♯ major
♩= 130
F
Cm/F
Gm/F
Fsus4
F
Cm/F
It's com-pli-cat - ed.
it al- ways is:
Gm/F
Fsus4
F
that's just the way it goes.
Feels like I've wait - ed so
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Sony/ATV Music Publishing(UK) Limited (50%)/Present Time Publishing(50%).
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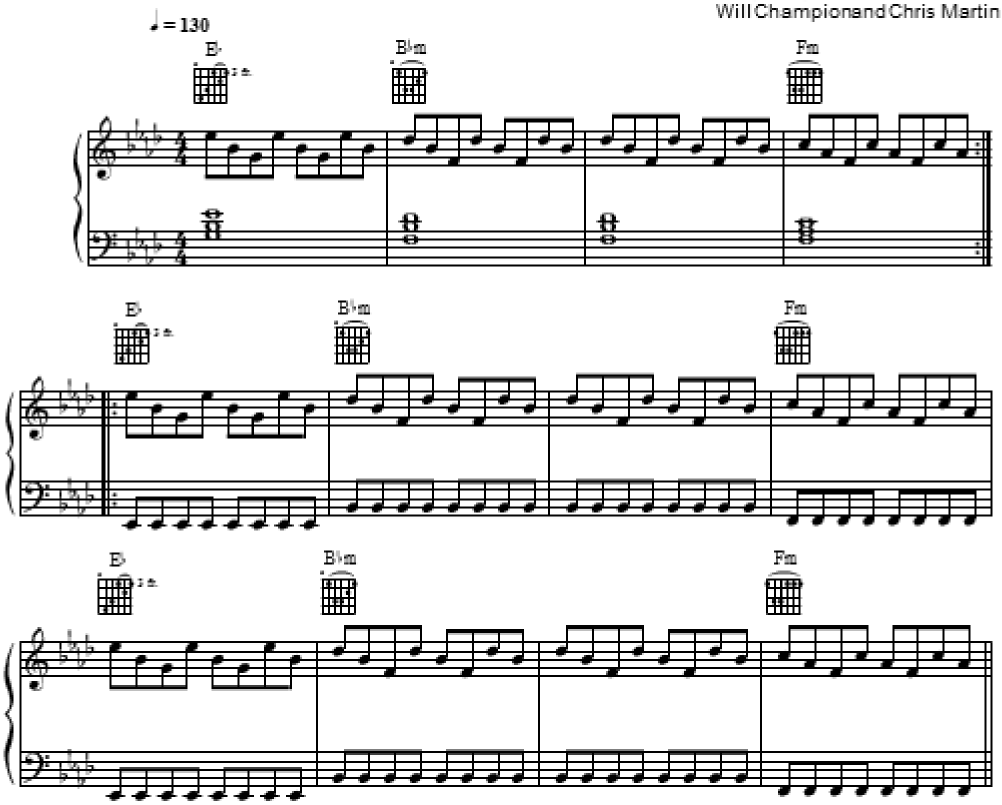
Appendix F

Clocks

Words and Music by
Guy Berryman, Jon Buckland,
Will Champion and Chris Martin



VI. REFERENCES

A, Andrew. "[Tutorial] The Theory Of & How To Read Music. No2. Basic Note Pitch Names." Web log post. *The Flipside Forum*. Simple Portal, 25 Jan. 2007. Web. 27 Nov. 2012. <http://www.theflipsideforum.com/index.php?topic=4144.0>.

"Clocks: Coldplay Digital Sheet Music." *Musicnotes.com: Download Sheet Music*. Musicnotes, Inc., n.d. Web. 28 Nov. 2012. <http://www.musicnotes.com/sheetmusic/mtdFPE.asp?ppn=MN0042978>.

"Famous Copyright Infringement Plagiarism Cases in Music." Famous Copyright Infringement Plagiarism Cases in Music. Weinhaus & Potashnick: Attorneys at Law, n.d. Web. 20 Sept. 2012. <http://www.fairwagelawyers.com/most-famous-music-copyright-infringment.html>.

"Fun Facts--Happy Birthday to You." *Fun Facts--Happy Birthday to You*. UNC School of Journalism, 21 Feb. 2003. Web. 28 Nov. 2012. <http://www.ibiblio.org/team/fun/birthday/>.

Grinstead, Charles M., and J. Laurie Snell. "Markov Chains." *Introduction to Probability*. 2nd ed. N.p.: n.p., 1998. 405-70. Print.

"Happy Birthday Sheet Music." *8notes.com*. 8notes, n.d. Web. 28 Nov. 2012. <http://8notes.com/scores/3029.asp>.

"Ice Ice Baby: Vanilla Ice Digital Sheet Music." *Musicnotes.com: Download Sheet Music*. Musicnotes, Inc., n.d. Web. 28 Nov. 2012. <http://www.musicnotes.com/sheetmusic/mtdFPE.asp?ppn=MN0065982&ref=google/>.

Leon, Steven J. "7.4 Matrix Norms and Condition Numbers." *Linear Algebra with Applications*. Eighth ed. Upper Saddle River, NJ: Prentice Hall, 2010. 403-09. Print.

"Row, Row, Row Your Boat Sheet Music." *8notes.com*. 8notes, n.d. Web. 28 Nov. 2012. <http://8notes.com/scores/6174.asp>.

Silver, Jason. "How to Read Sheet Music." *The Basics of Reading Music*. N.p., n.d. Web. 12 Sept. 2012. <http://readsheetmusic.info/readingmusic.shtml>.

"Under Pressure: Queen Digital Sheet Music." *Musicnotes.com: Download Sheet Music*. Musicnotes, Inc., n.d. Web. 28 Nov. 2012. <http://www.musicnotes.com/sheetmusic/mtdFPE.asp?ppn=MN0063595&ref=google/>.

“Vanilla Ice Interview Denies Under Pressure Rip Off on MTV 90s”. 07 December 2011. Online video clip. YouTube. Accessed on 27 November 2012.

Wagner, Adam. "David Guetta: When Love Takes Over Sounds like Coldplay: Clocks."*Sounds Just Like*. Sounds Just Like, n.d. Web. 28 Nov. 2012. <http://soundsjustlike.com/david-guettas-when-love-takes-over-sounds-like-coldplays-clocks/>.

"When Love Takes Over: David Guetta Digital Sheet Music." *Musicnotes.com: Download Sheet Music*. Musicnotes, Inc., n.d. Web. 28 Nov. 2012. <http://www.musicnotes.com/sheetmusic/mtdFPE.asp?ppn=MN0078832>.